Reweighted Regularized Sparse Recovery for DOA Estimation With Unknown Mutual Coupling

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***Abstract*— In this letter, the direction-of-arrival (DOA) estimation of uniform linear array under unknown mutual coupling is dealt with by proposing a reweighted regularized sparse recovery algorithm. The proposed method first formulates a block-sparse representation model without mutual coupling compensation and array aperture loss. Then, a reweighted l_1-norm minimization scheme is formulated to recover the block-sparse matrix, in which a weighted matrix is designed with a novel MUSIC-Like spectrum function for enhancing the sparsity of solution. Finally, the spatial spectrum of the recovered matrix is utilized to estimate DOAs. Due to using the whole array aperture and enhanced sparsity of solution, the proposed method can achieve superior performance than the existing regularized sparse recovery methods. Some simulation results are carried out to demonstrate the superiority of the proposed method.**

***Index Terms*— DOA estimation, uniform linear array, mutual coupling, block sparse recovery, weighted l_1-norm penalty.**

I. Introduction

THE estimation of direction-of-arrival (DOA) is a momentous basis in the field of array signal processing, which has been widely used in communication system, radar system, satellite navigation system and so on [1]. With the development of multiple-input multiple-output (MIMO) technique, especially applied in communication and radar systems, the MIMO systems achieve more degree of freedom (DOF) than conventional systems from the aspect of DOA estimation [2]. But with the increasing number of the antennas, the spacing between two elements will be decreased when the array aperture is fixed, which indicates that the strong mutual coupling would exist between the array elements. Therefore, the subspace based algorithms, such as MUSIC and ESPRIT algorithms [3], [4], degrade the performance or are unable to work.

Various array calibration algorithms based on subspace technique are studied for solving the DOA estimation problem of unknown mutual coupling [5]–[9]. In [5], it is pointed out that the mutual coupling matrix (MCM) of uniform linear array (ULA) is modeled as a banded symmetric Toeplitz matrix. Then in [6], an ESPRIT based algorithm with auxiliary array is proposed for robust DOA estimation by taking advantage of MCM. Following the idea of auxiliary array, the MUSIC-Like and ESPRIT-Like algorithms are proposed for angle estimation in MIMO systems [7], [8]. However, these methods lead to the array aperture loss. In [9], the special structure of MCM is transformed into a parameterized steering matrix for DOA estimation with the whole array aperture, then it combines the subspace technique to estimate DOA and achieves reasonable estimation performance.

In the past few years, the sparse recovery (SR) technique has attracted a lot of attention with array signal processing, and it is widely used in the estimation of DOA. In [10], a SR-based method called as l_1-SVD is investigated for DOA estimation with multiple signals, and then its extended versions, such as covariance vector based sparse representation methods [11], [12], are also investigated. All the simulation results have proved that these algorithms achieve superior estimation performance than subspace-based methods under lower SNR and/or limited number of snapshots. It is also noticed that the SR-based methods degrade the performance or cannot work with the unknown mutual coupling. To tackle the mentioned above issue, an effective sparse representation is investigated for DOA estimation by making use of the special structure of MCM [13], but the estimation performance is limited due to the aperture loss. In [14], a parameterized steering vector block sparse representation algorithm is proposed for DOA estimation by using the l_1-norm penalty with the unknown mutual coupling. Although this method avoids the aperture loss, the performance is also limited due to the fact that the l_1-norm penalty is an approximation of l_0-norm penalty in the regularized sparse recovery (RSR) strategy.

In this letter, a reweighted regularized sparse recovery algorithm is proposed for DOA estimation with unknown mutual coupling. A block-sparse representation model is formulated by parameterizing the steering vector without mutual coupling compensation and array aperture loss. Then a reweighted l_1-norm penalty scheme is proposed for DOA estimation by using the spatial spectrum of recovered block-sparse matrix, in which a novel MUSIC-Like spectrum function based on the parameterized steering vector is derived to design weighted matrix for enhancing the sparsity of solution. The proposed method can use the received data of whole array and achieve the enhanced sparsity of solution, therefore it exhibits superior performance than the existing regularized SR methods.

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Notation: $(\cdot)^{\mathrm{H}}$ and $(\cdot)^{\mathrm{T}}$ denote conjugate-transpose and transpose respectively. $diag\{\cdot\}$ is the diagonalization operation, and $\det\{\cdot\}$ is the determinant of the a matrix. $\mathbf{I}_K$ denotes a $K \times K$ identity matrix. $||\cdot||_0$, $||\cdot||_1$ and $||\cdot||_2$ denote the l_0 norm, l_1 norm and l_2 norm of a vector or matrix.

II. Data Model

Consider a uniform linear array (ULA) whose elements are placed on half-wavelength, and the array has M elements. Assuming that N uncorrected narrow band signals $s_n(t), (n = 1, 2, \cdots, N)$ impinging upon the array, where t denotes the time index. N sources with different and unknown DOAs are denoted as θ_n. Then the ideal steering vector at a DOA θ without mutual coupling can be shown as

$$\mathbf{a}(\theta) = [1, \rho(\theta), \rho(\theta)^2, \cdots, \rho(\theta)^{M-1}]^{\mathrm{T}}, \tag{1}$$

where $\rho(\theta) = \exp(j\pi\sin\theta)$ with $j = \sqrt{-1}$. Then the output of ideal array is expressed as

$$\bar{\mathbf{x}}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t), \tag{2}$$

where $\bar{\mathbf{x}}(t) = [\mathbf{x}_1(t), \mathbf{x}_2(t), \cdots, \mathbf{x}_M(t)]^{\mathrm{T}}$ represents the received data vector, and $\mathbf{A} = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \cdots, \mathbf{a}(\theta_N)]$ denotes the ideal steering matrix. $\mathbf{s}(t) = [s_1(t), s_2(t), \cdots, s_N(t)]^{\mathrm{T}}$ denotes the signal waveform. $\mathbf{n}(t)$ is the additive white Gaussian noise vector, whose mean and covariance matrix are zero and $\sigma^2\mathbf{I}_N$, respectively, where σ^2 denotes the noise power. In practice, the mutual coupling effect may exist between the closely spaced elements in array system. Generally, a banded symmetric Toeplitz matrix is adopted to represent the mutual coupling matrix (MCM) of ULA [5], whose form is shown as

$$\mathbf{Z} = \mathrm{Toeplitz}([1, z_1, \cdots z_{P-1}, \mathbf{0}_{1\times(M-P)}]), \tag{3}$$

where $\mathrm{Toeplitz}(\mathbf{r})$ denotes a returns the banded symmetric Toeplitz matrix formed from $\mathbf{r}$. $z_i, (i = 1, 2, \cdots, P-1)$ is the mutual coupling coefficients, which satisfies $0 < |z_{P-1}| < |z_{P-2}|, \cdots, |z_1| < z_0 = 1$. Then the steering vector in this case can be revised as

$$\bar{\mathbf{a}}(\theta) = \mathbf{Z}\mathbf{a}(\theta). \tag{4}$$

Thus, replacing the $\mathbf{a}(\theta)$ with $\bar{\mathbf{a}}(\theta)$, the output of array in Eq. (2) is converted into

$$\mathbf{x}(t) = \mathbf{Z}\mathbf{A}\mathbf{s}(t) + \mathbf{n}(t). \tag{5}$$

For L snapshots, the Eq. (5) is rewritten as

$$\mathbf{X} = \mathbf{Z}\mathbf{A}\mathbf{S} + \mathbf{N} = \mathbf{X} = \bar{\mathbf{A}}\mathbf{S} + \mathbf{N}, \tag{6}$$

where $\mathbf{X} = [\mathbf{x}(t_1), \mathbf{x}(t_1), \cdots, \mathbf{x}(t_L)]$ is the received data matrix. $\mathbf{S} = [\mathbf{s}(t_1), \mathbf{s}(t_2), \cdots, \mathbf{s}(t_L)]$, $\bar{\mathbf{A}} = \mathbf{Z}\mathbf{A}$ and $\mathbf{N} = [\mathbf{n}(t_1), \mathbf{n}(t_2), \cdots, \mathbf{n}(t_L)]$ are the signal waveform matrix, steering matrix with mutual coupling and additive noise matrix, respectively.

III. Reweighted Regularized Sparse Recovery for DOA Estimation

A. Parameterization of the Steering Vector and Block Sparse Representation Model

As shown in [6]–[8], a common way for removing the effect of unknown mutual coupling is that some elements are used to compensate the mutual coupling, which leads to array aperture loss. In order to avoid this drawback, the product of the steering vector and MCM is parameterized as [9]

$$\bar{\mathbf{a}}(\theta) = \mathbf{\Gamma}(\theta)\mathbf{g}(\theta), \tag{7}$$

where

$$\mathbf{\Gamma}(\theta) = \mathrm{blkdiag}\{\mathbf{\Gamma}_1, \mathbf{\Gamma}_2, \cdots, \mathbf{\Gamma}_3\}, \tag{8}$$

with $\mathbf{\Gamma}_1 = \mathrm{diag}\{1, \rho(\theta), \cdots, \rho(\theta)^{P-2}\}$, $\mathbf{\Gamma}_2 = [\rho(\theta)^{P-1}, \cdots, \rho(\theta)^{M-P}]^{\mathrm{T}}$, $\mathbf{\Gamma}_3 = \mathrm{diag}\{\rho(\theta)^{M-P+1}, \cdots, \rho(\theta)^{M-1}\}$, and

$$\mathbf{g}(\theta) = [\mu_1(\theta), \cdots, \mu_{P-1}(\theta), \nu(\theta), \omega_1(\theta), \cdots, \omega_{P-1}(\theta)]^{\mathrm{T}}, \tag{9}$$

with $\mu_i(\theta) = 1 + \sum_{i=1}^{P-1} z_i\rho(\theta)^i + \sum_{i=1}^{k-1} z_i\rho(\theta)^{-i}$, $\omega_i(\theta) = 1 + \sum_{i=1}^{P-1} z_i\rho(\theta)^{-i} + \sum_{i=1}^{P-k-1} z_i\rho(\theta)^i$ and $\nu(\theta) = 1 + \sum_{i=1}^{P-1} z_i(\rho(\theta)^i + \rho(\theta)^{-i})$. As discussing in [9], $\nu(\theta)$ may take a zero value with some very special cases, however it is a small probability event. In general, $\nu(\theta)$ is assumed to be nonzero in the following section. Then substituting the Eq. (7) back into Eq. (6), the Eq. (6) can be revised as

$$\begin{aligned}\mathbf{X} &= \bar{\mathbf{A}}\mathbf{S} + \mathbf{N} = \mathbf{B}\mathbf{\Lambda}\mathbf{S} + \mathbf{N} \\ &= \mathbf{B}\bar{\mathbf{S}} + \mathbf{N},\end{aligned} \tag{10}$$

where

$$\mathbf{B} = [\mathbf{\Gamma}(\theta_1), \mathbf{\Gamma}(\theta_2), \cdots, \mathbf{\Gamma}(\theta_N)], \tag{11}$$

$$\mathbf{\Lambda} = \mathrm{blkdiag}\{\mathbf{g}(\theta_1), \mathbf{g}(\theta_2), \cdots, \mathbf{g}(\theta_N)\}, \tag{12}$$

and $\bar{\mathbf{S}} = \mathbf{\Lambda}\mathbf{S}$. According to Eq. (10), it can be seen that $\mathbf{B}$ is composed with $\mathbf{\Gamma}(\theta_n)(n = 1, 2, \cdots, N)$ and becomes a new steering matrix depended on the information of DOAs, which indicates that the mutual coupling does not affect the new steering matrix. $\bar{\mathbf{S}}$ is a new signal matrix after this operation. In addition, it is also noticed that for each source, the corresponding array manifold and signal data have block structures in Eq. (10), which is very different with the steering vector and signal waveform vector in Eq. (6). Following the idea of sparse representation, the Eq. (10) is converted into a block sparse representation model via constructing an over complete dictionary via discretizing the the spatial domain. Let $\mathbf{\Theta} = [\bar{\theta}_1, \bar{\theta}_2, \cdots, \bar{\theta}_K]$ be the spatial sampling grid, and all the possible DOAs are on the sampling grids, where K denotes the total number of sampling grids. Then an over-complete dictionary corresponding to the sampling grids is constructed as

$$\bar{\mathbf{B}} = [\mathbf{\Gamma}(\bar{\theta}_1), \mathbf{\Gamma}(\bar{\theta}_2), \cdots, \mathbf{\Gamma}(\bar{\theta}_K)], \tag{13}$$

where each potential source has $M \times (2P-1)$ block steering matrix $\mathbf{\Gamma}(\bar{\theta}_k)(k = 1, 2, \cdots, K)$. On the other hand, the single value decomposition (SVD) technique is usually adopted in the procedure of sparse recovery for reducing the computational burden and the sensitivity to noise. Applying the SVD to the received data in Eq. (10) yields

$$\mathbf{X} = \mathbf{U}\mathbf{\Delta}\mathbf{V}, \tag{14}$$

where $\mathbf{\Delta}$ is a diagonal matrix, and its diagonal elements correspond to the singular values, and $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices. Let $\mathbf{V}_s$ be composed of N columns of $\mathbf{V}$ corresponding to the N largest singular values, which is also named as signal subspace. Then multiplying $\mathbf{V}_s$ with $\mathbf{X}$ yields

$$\bar{\mathbf{X}} = \mathbf{B}\mathbf{S}_v + \mathbf{N}_v, \tag{15}$$

where $\bar{\mathbf{X}} = \mathbf{X}\mathbf{V}_s$, $\mathbf{S}_v = \bar{\mathbf{S}}\mathbf{V}_s$ and $\mathbf{N}_v = \mathbf{N}\mathbf{V}_s$. Then using the over complete dictionary in Eq. (13), the received data in Eq. (15) is sparsely represented as

$$\bar{\mathbf{X}} = \bar{\mathbf{B}}\mathbf{S}_{\bar{\theta}} + \mathbf{N}_v, \tag{16}$$

where $\mathbf{S}_{\bar{\theta}} = [\mathbf{S}_{\bar{\theta}_1}, \mathbf{S}_{\bar{\theta}_2}, \cdots, \mathbf{S}_{\bar{\theta}_K}]^{\mathrm{T}} \in \mathbb{C}^{KQ\times N}$ is a block sparse matrix, and the kth block matrix $(\mathbf{S}_{\bar{\theta}})_k$ corresponds to the $(Qk-Q+1)$th to (Qk)th rows of $\mathbf{S}_{\bar{\theta}}$, where $Q = 2P-1$. For a true DOA, the corresponding block is nonzero, which means that the block sparse matrix $\mathbf{S}_{\bar{\theta}}$ only has N nonzero blocks. Therefore, the block sparse spatial spectrum of $\mathbf{S}_{\bar{\theta}}$ is used to estimate the DOAs. The following section will show how to recover the block sparse matrix $\mathbf{S}_{\bar{\theta}}$ for DOA estimation.

B. Weighted l_1-SVD Method for DOA Estimation

In order to recover the block sparse matrix in Eq. (16), the l_0-norm penalty is selected as an ideal measure of sparsity, and its corresponding optimization problem can be formulated as the following form

$$\min \parallel \mathbf{S}_{\bar{\theta}}^{l_2} \parallel_0 \quad \text{s.t.} \quad \parallel \bar{\mathbf{X}} - \bar{\mathbf{B}}\mathbf{S}_{\bar{\theta}} \parallel_2 \leq \eta, \tag{17}$$

where η is regularization parameter, which controls the upper bound of the fitting error. $\mathbf{S}_{\bar{\theta}}^{l_2} = [\mathbf{S}_{\bar{\theta}_1}^{l_2}, \mathbf{S}_{\bar{\theta}_2}^{l_2}, \cdots, \mathbf{S}_{\bar{\theta}_K}^{l_2}]^{\mathrm{T}}$, and $\mathbf{S}_{\bar{\theta}_K}^{l_2}$ is the l_2-norm value of $\mathbf{S}_{\bar{\theta}_k}$. $\mathbf{S}_{\bar{\theta}_k}$ is the kth block of $\mathbf{S}_{\bar{\theta}}$, which is composed of the $(Qk-Q+1)$th to the (Qk)th rows of $\mathbf{S}_{\bar{\theta}}$. Although the l_0-norm constrained scheme can achieve the optimal recovery performance, it is NP-hard and mathematically intractable. Following the idea in [14], the l_1-norm is used to replace the l_0-norm for relaxing the constrain. Then the l_1-norm minimization problem for DOA estimation is expressed as

$$\min \parallel \mathbf{S}_{\bar{\theta}}^{l_2} \parallel_1 \quad \text{s.t.} \quad \parallel \bar{\mathbf{X}} - \bar{\mathbf{B}}\mathbf{S}_{\bar{\theta}} \parallel_2 \leq \eta, \tag{18}$$

The Eq. (18) is a convex optimization problem, and the CVX software is adopted to solve it [15]. However, it has been shown that the l_1-norm penalizes the large entries more heavier than small entries, which leads the limited recovery performance [16]. In order to enforce the sparsity of the solution, a weighted matrix, based on the orthogonality between the over-complete dictionary and the noise subspace, is designed for reweighting l_1-norm minimization problem. Let the over-complete dictionary be divided into two submatrices along the columns, which is shown as $\bar{\mathbf{B}} = [\bar{\mathbf{B}}_1, \bar{\mathbf{B}}_2]$, where $\bar{\mathbf{B}}_1$ is assumed to consist of N block steering matrices corresponding the true DOAs, and $\bar{\mathbf{B}}_2$ is made up of the residual block steering matrices. And the noise subspace $\mathbf{U}_n$ is consisted of the $M-N$ columns of $\mathbf{U}$ associated with $M-N$ smaller singular values. Then a novel MUSIC-Like spectrum function is constructed as

$$\bar{w}_k = \det\{\mathbf{\Gamma}(\bar{\theta}_k)^{\mathrm{H}}\mathbf{U}_n\mathbf{U}_n^{\mathrm{H}}\mathbf{\Gamma}(\bar{\theta}_k)\}, \tag{19}$$

Utilizing the spatial spectrum in Eq. (19), a weighted matrix is constructed as

$$\mathbf{W} = \mathrm{diag}\{\mathbf{w}\}, \tag{20}$$

where $\mathbf{w} = [w_1, w_2, \cdots, w_K] = [\mathbf{w}_1, \mathbf{w}_2]$ with $w_k = \bar{w}_k/\max\{\bar{w}_1, \bar{w}_2, \cdots, \bar{w}_K\}$. For the steering matrix $\bar{\mathbf{B}}_1$ of true DOAs, its corresponding weights in $\mathbf{w}_1$ is more smaller than the residual weights in $\mathbf{w}_2$. The weights in $\mathbf{w}_1$ satisfies $\mathbf{w}_1 \rightarrow 0$ when the number of snapshots is infinite. When applying this weighted matrix to l_1-norm minimization problem, the small entries, who are more close to zero, are punished by large weights, and those larger entries are reserved by small weights. This operation with multiple measure vector (MMV) can achieve the similar methodology of the iterative reweighted l_1-norm minimization for single measure vector (SMV) issue in [16]. Thus, this weighted matrix can enforce the sparsity of the solution. Then a reweighted optimization model based on l_1-norm is given as follows

$$\min \parallel \mathbf{W}\mathbf{S}_{\bar{\theta}}^{l_2} \parallel_1 \quad \text{s.t.} \quad \parallel \bar{\mathbf{X}} - \bar{\mathbf{B}}\mathbf{S}_{\bar{\theta}} \parallel_2 \leq \eta, \tag{21}$$

Eq. (21) is a second order cone (SOC) programming problem, which can be solved by the optimization software packages, such as CVX. Finally, the DOAs can be achieved from the spatial spectrum of $\mathbf{S}_{\bar{\theta}}^{l_2}$.

Remark: The selection of regularization parameter η is an important issue for the reweighted l_1-norm minimization problem. According to the asymptotic chi-square distribution of $||\mathbf{N}_v||_2^2$, the regularization parameter η has been suggested to choose the upper bound of $||\mathbf{N}_v||_2^2$ with 99% confidence interval [10]. And the main computation burden of the proposed methods is to perform the weighted matric and solving SOC programming in the Eq. (21). It requires $O\{KQ^4(M-N)(M+Q)+(NKQ)^3\}$ flops, and the complexity of l_1-norm penalty in [14] is $O\{(NKQ)^3\}$. Thus the computational complexity of the proposed method is higher than the l_1-norm penalty in [14]. But the proposed method achieves enhanced sparsity solution and provides superior estimation performance.

IV. Simulation Results

In this part, simulation experiments are given to demonstrate the DOA estimation performance of the proposed reweighted regularized sparse recovery algorithm. The l_1-SVD method in [13], the block sparse recovery (BSR) method in [14] and the CRB in [17] are chosen for comparing with our proposed method. The number of elements of ULA with half-wavelength spacing is set as $M = 10$. The number of signals is assumed to be known, and two uncorrected signals are from different DOAs, which are denoted as $\theta_1 = 0.8°$ and $\theta_1 = -15.5°$. It is assumed that $P = 3$ and the mutual coupling coefficients are $z_1 = 0.4864 - 0.4776j$ and $z_2 = 0.10 - 0.1706j$. The spatial sampling grid is set as $0.02°$

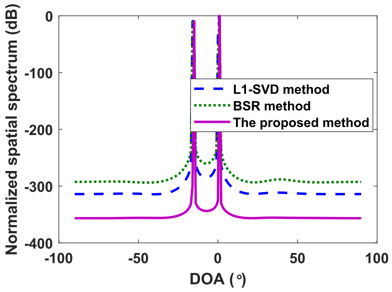
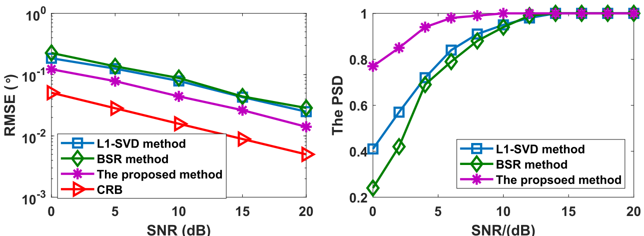
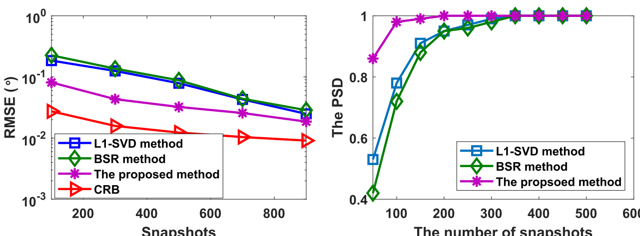


Fig. 1. Normalized spatial spectrum of three different methods.



(a) RMSE versus SNR (b) PSD versus SNR

Fig. 2. The comparison of RMSE/PSD versus SNR.



(a) RMSE versus snapshot (b) PSD versus snapshot

Fig. 3. The comparison of RMSE/PSD versus snapshot

from $-90°$ to $90°$, and the root mean square error (RMSE) is utilized to compare the performance of these algorithms.

Fig. 1 shows the normalized spatial spectrum of three different algorithms, where the snapshot number is selected as 300 and SNR $=$ 10dB is used. It is shown that the proposed method has more sharp peaks and lower sidelobe than the other methods, which indicates that our method has advantages on estimation accuracy and spatial resolution.

Fig. 2 depicts the comparison of RMSE/PSD versus SNR with three different algorithms, where the snapshot number is selected as 300 and 200 Monte-Carlo simulations are used. The probability of successful detection (PSD) is defined as that the estimation error satisfies $|\theta_n - \hat{\theta}_n| < 0.5°$, where $\hat{\theta}_n$ is the estimated value of θ_n. As shown in Fig.2, our proposed method is more accurate and has a lower RMSE and higher PSD than l_1-SVD and BSR methods. The main reason is that the proposed method uses the reweighted l_1-norm to enhance the sparsity of solution and is without the loss of array aperture. But there is a clear gap between the RMSE of DOA obtained by SR-based methods and CRB, and this is because all of these method only achieve the suboptimal solution.

Fig.3 depicts the comparison of RMSE/PSD versus snapshot number, where SNR is selected as 10dB and 200 Monte-Carlo simulations are utilized. As the snapshot number increase, the estimation performance and PSD of all methods are improved. On the other hand, our method is still superior to the l_1-SVD and BSR methods in both RMSE and PSD for all snapshot sizes. We also noticed that l_1-SVD and BSR methods have similar performance with different snapshot number.

V. Conclusion

In this letter, a reweighted regularized sparse recovery algorithm is proposed for DOA estimation with unknown mutual coupling. In our proposed method, the DOA is obtained from a reweighted l_1-norm minimization based block sparse recovery scheme via parameterizing steering vector. The proposed method takes advantage of the whole array aperture and enhances sparsity of solution, achieving better DOA estimation performance without mutual coupling compensation. Simulation experiments verify that the proposed method has remarkable advantages over the existing regularized sparse recovery methods.

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