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Robust Weighted Subspace Fitting for DOA Estimation via Block Sparse Recovery

Dandan Meng, Xianpeng Wang, *Member, IEEE*, Mengxing Huang, Liantian Wan , *Member, IEEE* and Bin Zhang

***Abstract*—In this letter, a novel robust block sparse recovery algorithm by using the weighted subspace fitting (WSF) is proposed to deal with the direction-of-arrival (DOA) problem under the condition of unknown mutual coupling. Firstly, a novel block sparse representation signal model based on the WSF is established to settle the effect of unknown mutual coupling. Then, the sparse constraint problem is investigated, and a regularization criterion between the sparsity penalty and subspace fitting error is given. Finally, the DOA estimation problem can be converted into a block sparse recovery problem. Some experimental results are carried out to prove the performance of proposed method in the case of unknown mutual coupling.**

***Index Terms*—DOA estimation, weighted subspace fitting, unknown mutual coupling, robust block sparse recovery,**

I. INTRODUCTION

In the last few decades, array signal processing has been widely encountered and rapidly developed in seismic detection, radar and other fields [1]. Its main purpose is to enhance the useful signals and suppress interference and noise, and extract the useful signal characteristics and information. And the direction-of-arrival (DOA) estimation issue is one of the most important topics, which has been attracted extensive attention [2]. The traditional DOA estimation algorithms such as: the multiple signals classification (MUSIC) [3] and estimating signal parameter via rotational invariance techniques (ESPRIT) algorithm [4] and so on, which are classed as subspace-based algorithms. However, these methods require enough number of the data and/or reasonable high SNR to achieve the desired performance. Recently, with the development of compressive sensing (CS) technology, the l_1-singular value decomposition (SVD) [5] and sparse Bayesian learning (SBL) algorithm [6] have been proposed to solve the limitation of subspace-based algorithms, which are called as the sparse signal recovery (SSR)-based methods. And a large number of studies have demonstrated that the SSR-based algorithms get better estimation performance than the subspace-based, especially at low SNR or/and limited snapshots [7-10].

However, in reality, many studies have found that because of the interaction of electromagnetic fields, the mutual coupling effect is an inherent characteristic between the closely antennas [11-13]. Thus, the structure of ideal steering vector may be destroyed severely. Then the above algorithms depended on the ideal array manifold will fail to work in the case of unknown mutual coupling. To solve the above problem, a lot of researches have been done [14-20]. For the subspace framework, the ESPRIT-Like method in [14] was investigated by using the auxiliary array to estimate DOA. But it has the same limitation with subspace-based method, and leads to array aperture loss. To settle this problem, a novel subspace method based on the block representation data model without array aperture loss was presented in [15]. But it still belongs to the subspace methods. On the other hand, for the SSR framework, an l_1-SVD algorithm was presented by utilizing the characteristic of mutual coupling matrix (MCM) [16]. But it requires large-scale array to ensure the performance due to the array aperture loss. Then, a modified l_1-SVD method by using a block representation data model was constructed to settle the problem of array aperture loss [17]. But the sparse solution of l_1-norm is only an approximation of the l_0-norm penalty, which has the limitation of the estimation performance.Then a weighted matrix based on the MUSIC-Like function is constructed to solve the above problem [12], but its estimation performance is based on the received data. Then a reweighted nuclear norm minimization method is constructed [18], but this received data fitting model is not optimal. And the sparse Bayesian method based on a comprehensive array output model that is applicable to each type of the typical array perturbations is formulated in [19], but its root mean square error (RMSE) is much larger than the Cramer-Rao bound (CRB) under low SNR.

In this letter, a robust weighted subspace fitting (WSF) DOA estimation under the condition of unknown mutual coupling via block sparse recovery in uniform linear array (ULA) is proposed. A new block spars representation signal model without array aperture loss is firstly constructed based on the WSF, which avoids the mutual coupling effects. Then, the sparse constraint problem is investigated for the optimal weighted subspace fitting algorithm, and the upper limit of fitting error is given. Finally, the DOA estimation issue is transformed into block sparse recovery (BSR) problem. Some simulation results are given to prove the high performance of the proposed algorithm.

Notation: $\mathrm{blkdiag}\{\cdot\}$ denotes the block diagonalization

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D. Meng, X. Wang and M. Huang are with the State Key Laboratory of Marine Resource Utilization in South China Sea and College of Information Science and Technology, Hainan University, Haikou 570228, China, (E-mail:wxpeng1986@126.com).

L. Wan is with The School of Software, Dalian University of Technology, Dalian, 116024, China.

B. Zhang is with The Department of Mechanical Engineering, Kanagawa University, 3-27-1 Rokkakubashi, Kanagawa-ku, Yokohama 221-8686, Japan.

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operation. $(\cdot)^{\mathrm{H}}$ and $(\cdot)^{\mathrm{T}}$ represent conjugate-transpose and transpose respectively. $||\cdot||_0$, $||\cdot||_1$, $||\cdot||_2$ and $||\cdot||_{\mathrm{F}}$ denote the l_0 norm, l_1 norm, l_2 norm and Frobenius norm, respectively. $\mathbf{I}_K$ is the $K \times K$ identity matrix.

II. Data Model

A. Problem Formulation

Suppose a ULA, which consists of M array elements with arbitrary directivity. The spacing d between the adjacent antennas equals to half-wavelength. Suppose P far-field signals, which are from different directions $\boldsymbol{\theta} = [\theta_1, \theta_2, \cdots, \theta_P]$ incident on the array. However, in practice, a large number of studies have found that due to the interaction of electromagnetic fields, there will be mutual coupling between the closely spaced antennas, which is considered here. Then the data output in the presence of unknown mutual coupling could be modeled as

$$\mathbf{x}(t) = \sum_{p=1}^{P} \mathbf{T}\mathbf{a}(\theta_p)s_p(t) + \mathbf{n}(t) = \mathbf{T}\mathbf{A}\mathbf{s}(t) + \mathbf{n}(t), \tag{1}$$

where $\mathbf{x}(t) = [x_1(t), x_2(t), \cdots, x_M(t)]^{\mathrm{T}} \in^{M\times 1}$ represents the array of received data. $\mathbf{A} = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \ldots, \mathbf{a}(\theta_P)] \in \mathbb{C}^{M\times P}$ is the ideal array manifold, which has no mutual coupling effects. And $\mathbf{a}(\theta_p) = [1, \nu(\theta_p), \cdots, \nu(\theta_p)^{M-1}]^{\mathrm{T}}$ $(p = 1, 2, \cdots, P)$ denotes the steering vector, where $\nu(\theta_p) = e^{j2\pi f c^{-1} d \sin\theta_p}$, where c is the propagation speed, $j = \sqrt{-1}$. $\mathbf{s}(t)$ is the $P \times 1$ signal vector with $\mathbf{s}(t) = [s_1(t), s_2(t), \ldots, s_P(t)]^{\mathrm{T}}$, and $\mathbf{n}(t) \sim CN(0, \sigma^2\mathbf{I})$ represents the $M \times 1$ independent and narrow-band white noise vector, and σ^2 denotes the noise power. And the matrix $\mathbf{T}$ represents the MCM, which can be modeled by a banded Toeplitz structure matrix [13]:

$$\mathbf{T} = \mathrm{Toepliz}([1, t_1, \cdots t_{K-1}, \mathbf{0}_{1\times(M-K)}]), \tag{2}$$

where $\{t_i\}_{i=1}^{K-1}$ are the normalized mutual coupling coefficients, and it satisfies $0 < \left|t_{(K-1)}\right| < \left|t_{(K-2)}\right| <, \cdots, |t_2| < |t_1| < 1$, and K represents the number of non-zero mutual coupling coefficients in the first row in MCM. That is to say, when the spacing between the two array antennas is smaller than $(K-1)d$, the mutual coupling effects cannot be overlooked.

Then from Eq.(1), the covariance matrix can be calculated as

$$\bar{\mathbf{R}} = \mathrm{E}\{\mathbf{x}(\mathrm{t})\mathbf{x}^{\mathrm{H}}(\mathrm{t})\} = \mathbf{T}\mathbf{A}\mathbf{R}_{\mathrm{s}}\mathbf{A}^{\mathrm{H}}\mathbf{T}^{\mathrm{H}} + \sigma^2\mathbf{I}_{\mathrm{M}}, \tag{3}$$

where $\mathbf{R}_s$ denotes the covariance matrix of $\mathbf{s}(t)$, and the rank of matrix $\mathbf{R}_s$ is defined as P' , which depends on the correlation between the signals. In practice, the data is got from limited snapshots number, through the above series of analysis, now we consider L snapshots number, the Eq.(3) can be replaced by

$$\mathbf{R} = \frac{1}{L}\sum\nolimits_{t=1}^{L} \mathbf{x}(t)\mathbf{x}^{\mathrm{H}}(t), \tag{4}$$

Then the eigenvalue decomposition of $\mathbf{R}$ can be represented as

$$\mathbf{R} = \sum\nolimits_{i=1}^{M} \mu_i \mathbf{e}_i \mathbf{e}_i^{\mathrm{H}} = \mathbf{E}_s\boldsymbol{\Sigma}_s{\mathbf{E}_s}^{\mathrm{H}} + \mathbf{E}_n\boldsymbol{\Sigma}_n{\mathbf{E}_n}^{\mathrm{H}}, \tag{5}$$

where $\mu_1 \geq \mu_2 \geq \cdots \geq \mu_{P'} \geq \mu_{P'+1} = \cdots \mu_M$ denotes the eigenvalues, $\boldsymbol{\Sigma}_s = \mathrm{diag}\{\mu_1, \mu_2, \cdots, \mu_{P'}\}$, $\boldsymbol{\Sigma}_n = \mathrm{diag}\{\mu_{\mathrm{P}'+1}, \mu_{\mathrm{P}'+2}, \cdots, \mu_{\mathrm{M}}\}$. When the signals are coherent, P' is less than P, otherwise P' equals to P. And the matrices $\mathbf{E}_s$, $\mathbf{E}_n$ denote the eigenvectors corresponding to the P' largest eigenvalues and $M - P'$ smallest eigenvalues, which are named signal subspace and noise subspace, respectively [22].

B. Parameterization of the Steering Matrix with Unknown Mutual Coupling

Using the principle of subspace, the signal subspace $\mathbf{E}_s$ stays in the same subspace of array manifold matrix [21]. There is a full rank matrix $\tilde{\mathbf{H}}$ that satisfies $\mathbf{E}_s = \mathbf{T}\mathbf{A}\tilde{\mathbf{H}}$. However, due to the unknown mutual coupling coefficient, the $\tilde{\mathbf{H}}$ cannot be estimated immediately. On the other hand, from Eq.(1), it is indicated that the structure of steering vector is destroyed under the unknown mutual coupling conditions, which also can be redefined as follows:

$$\tilde{\mathbf{a}}(\theta) = \mathbf{T}\mathbf{a}(\theta), \tag{6}$$

where $\tilde{\mathbf{a}}(\theta)$ is the coupled steering vector. Now, the vector $\tilde{\mathbf{a}}(\theta)$ can be parameterized with the decoupled DOAs and the mutual coupling coefficient, which is rewritten as [15]

$$\tilde{\mathbf{a}}(\theta) = \mathbf{T}\mathbf{a}(\theta) = \boldsymbol{\Psi}(\theta)\boldsymbol{\Delta}(\theta), \tag{7}$$

where

$$\boldsymbol{\Psi}(\theta) = \mathrm{blkdiag}\{\boldsymbol{\Psi}_1, \boldsymbol{\Psi}_2, \boldsymbol{\Psi}_3\}, \tag{8}$$

and

$$\boldsymbol{\Delta}(\theta) = [\varsigma_1(\theta), \cdots, \varsigma_{K-1}(\theta), \tau(\theta), \phi_1(\theta), \cdots, \phi_{K-1}(\theta)]^{\mathrm{T}}, \tag{9}$$

where $\boldsymbol{\Psi}_1 = \mathrm{diag}\{1, \nu(\theta), \cdots, \nu(\theta)^{K-2}\} \in^{(K-1)\times(K-1)}$, $\boldsymbol{\Psi}_2 = [\nu(\theta)^{K-1}, \cdots \nu(\theta)^{M-K}]^{\mathrm{T}} \in^{(M-2K+2)}$, $\boldsymbol{\Psi}_3 = \mathrm{diag}\{\nu(\theta)^{M-K+1}, \ldots, \nu(\theta)^{M-1}\} \in^{(K-1)\times(K-1)}$. And $\varsigma_z(\theta) = 1 + \sum_{i=1}^{K-1} t_i\nu(\theta)^i + \sum_{i=1}^{z-1} t_i\nu(\theta)^{-i}$, $\phi_z(\theta) = 1 + \sum_{i=1}^{K-1} t_i\nu(\theta)^{-i} + \sum_{i=1}^{K-z-1} t_i\nu(\theta)^i$ and $\tau(\theta) = 1 + \sum_{i=1}^{K-1} t_i(\nu(\theta)^i + \nu(\theta)^{-i})$, $z = 1, 2, \cdots, K-1$. As described in [21], $\tau(\theta)$ equals zero with a small probability. So $\nu(\theta)$ is assumed to be nonzero in this letter. Then, combining Eq.(7) and Eq.(1), and Eq.(1) could be transformed to

$$\begin{aligned} \mathbf{x}(t) &= \mathbf{T}\mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) = \bar{\mathbf{A}}\boldsymbol{\Lambda}\mathbf{s}(t) + \mathbf{n}(t) \\ &= \bar{\mathbf{A}}\bar{\mathbf{s}}(t) + \mathbf{n}(t), \end{aligned} \tag{10}$$

where

$$\bar{\mathbf{A}} = [\boldsymbol{\Psi}(\theta_1), \boldsymbol{\Psi}(\theta_2), \cdots, \boldsymbol{\Psi}(\theta_P)], \tag{11}$$

$$\boldsymbol{\Lambda} = \mathrm{blkdiag}\{\boldsymbol{\Delta}(\theta_1), \boldsymbol{\Delta}(\theta_2), \cdots, \boldsymbol{\Delta}(\theta_P)\}, \tag{12}$$

where $\bar{\mathbf{A}} \in^{M\times PQ}$ represents the new parameterized array manifold matrix depends on the DOAs without the mutual coupling coefficient, which indicated that the effect of mutual coupling is removed. Then we define $Q = 2K - 1$, and it is indicated that $\boldsymbol{\Lambda}$ is a block diagonal matrix, defining $\bar{\mathbf{s}}(t) = \boldsymbol{\Lambda}\mathbf{s}(t)$, which denotes the new signal matrix, and the $(Q(p-1)+1)$ th to (Qp) th rows of signal $\bar{\mathbf{s}}(t)$ corresponds to the pth value of $\mathbf{s}(t)$.

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III. Weighted Subspace Fitting for DOA estimation via Block Sparse recovery

According to the analysis in the above section and the basic principles of linear algebra, the new array manifold matrix $\bar{\mathbf{A}}$ still spans the same subspace as the signal subspace $\mathbf{E}_s$, which satisfies:

$$\mathbf{E}_s = \bar{\mathbf{A}}\mathbf{\Lambda}\tilde{\mathbf{H}} = \bar{\mathbf{A}}\bar{\mathbf{H}}, \tag{13}$$

where $\bar{\mathbf{H}} = \mathbf{\Lambda}\tilde{\mathbf{H}}$ denotes a $PQ\times P'$ full rank block matrix, and the $(Q(p-1)+1)$ th to(Qp) th rows of matrix $\bar{\mathbf{H}}$ corresponding to the pth row of matrix $\tilde{\mathbf{H}}$, and $p = 1, 2, \cdots, P$. However, due to the existence of noise in the received data, Eq. (13) will not be valid. According to [11], the above problem can be transformed into the weighted subspace fitting in the sense of least squares by solving Eq.(14) as following [21]:

$$\hat{\boldsymbol{\theta}} = \arg\min \left\|\mathbf{E}_s\mathbf{W} - \bar{\mathbf{A}}(\boldsymbol{\theta}_0)\mathbf{H}\right\|, \tag{14}$$

where $\mathbf{W}$ denotes the weighted matrix which varies with different methods. By separating variables $\mathbf{H}$, the subspace fitting with optimal weighted matrix can be obtained by solving Eq.(15) as:

$$\hat{\boldsymbol{\theta}} = \arg\min \mathrm{tr}\{\mathbf{P}_{\bar{\mathbf{A}}}^{\perp}\mathbf{E}_s\mathbf{W}_{opt}{\mathbf{E}_s}^{\mathrm{H}}\} = \arg\min \mathbf{Z}(\boldsymbol{\theta}_0), \tag{15}$$

where $\mathbf{P}_{\bar{\mathbf{A}}}^{\perp} = \bar{\mathbf{A}}(\bar{\mathbf{A}}^{\mathrm{H}}\bar{\mathbf{A}}^{-1})\bar{\mathbf{A}}^{\mathrm{H}}$ is the orthogonal projection matrix of $\bar{\mathbf{A}}$, and $\mathbf{W}_{opt} = (\mathbf{\Sigma}_s - \sigma^2\mathbf{I}_K)^2{\mathbf{\Sigma}_s}^{-1}$ is the optimal weighted matrix which has been proved in [21].

In order to investigate the subspace fitting problem from the perspective of sparse signal recovery, an over-complete dictionary $\hat{\mathbf{A}} = [\mathbf{\Psi}(\theta_1), \mathbf{\Psi}(\theta_2), \cdots, \mathbf{\Psi}(\theta_N)]$ is established. $\theta_n(n = 1, 2, \cdots, N)$ denotes the potential grid points, and the desired DOAs fall into this spatial scope. Then, the sparse representation of $\mathbf{E}_s\mathbf{W}_{opt}$ can be represented as

$$\mathbf{E}_s\mathbf{W}_{opt} = \hat{\mathbf{A}}\hat{\mathbf{H}}, \tag{16}$$

where $\hat{\mathbf{H}}$ is the block sparse matrix, and the non-zero block of $\hat{\mathbf{H}}$ correspond to the true DOAs, which has the same value with those of matrix $\bar{\mathbf{H}}$. Then, the DOA estimation problem is solved by finding that the position of non-zero blocks in $\hat{\mathbf{H}}$ correspond to the angle values in the over-complete dictionary $\hat{\mathbf{A}}$.

Then, the recovery of block sparse matrix $\hat{\mathbf{H}}$ can be transformed into the following l_0-norm constrained optimization problem:

$$\min\left\|\hat{\mathbf{H}}^{\circ}\right\|_0, \quad \text{s.t.} \quad \mathbf{E}_s\mathbf{W}_{opt} = \hat{\mathbf{A}}\hat{\mathbf{H}}, \tag{17}$$

where $\hat{\mathbf{H}}^{\circ} = [\hat{h}_1^{\circ}, \hat{h}_2^{\circ}, \cdots, \hat{h}_N^{\circ}]$ denotes a new vector for convenience, whose the nth value corresponding to the l_2-norm of the $(Q(n-1)+1)$th to (Qn) th rows in $\hat{\mathbf{H}}$. Thus, the non-zero value of $\hat{\mathbf{H}}^{\circ}$ correspond to the real DOAs and has the same block sparsity as matrix $\hat{\mathbf{H}}$. Meanwhile, through the discussion in [14], the Eq.(17) is NP-hard. At present, the commonly used method is to replace l_0-norm with l_1-norm. What's more, the existence of fitting error must be considered in finite snapshots. Based on the above discussion, the recovery of matrix $\hat{\mathbf{H}}$ in Eq.(17) can be reconverted to the following joint sparse recovery problem:

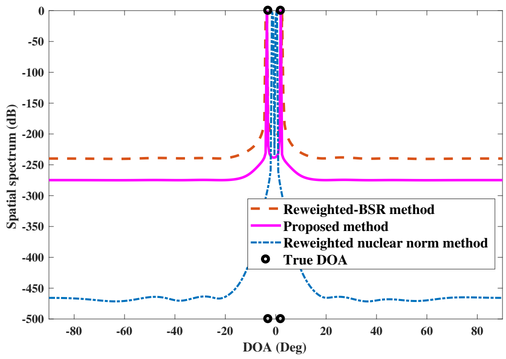

Fig. 1. Spatial spectrum for incoherent closely spaced signals

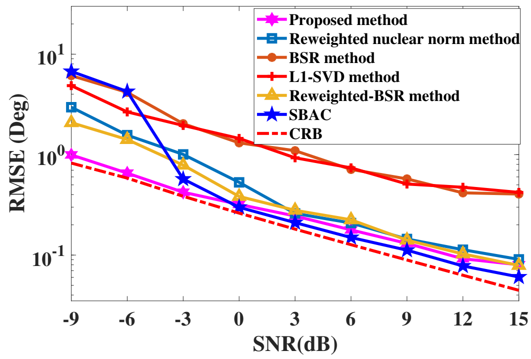

Fig. 2. RMSE versus SNR for incoherent closely spaced signals

$$\min\left\|\hat{\mathbf{H}}^{\circ}\right\|_1, \quad \text{s.t.} \quad \left\|\mathbf{E}_s\mathbf{W}_{opt} - \hat{\mathbf{A}}\hat{\mathbf{H}}\right\|_{\mathrm{F}} \le \delta, \tag{18}$$

where parameter δ denotes the upper bound of subspace fitting error. According to [22], the second-order cone (SOC) programming can be utilized to calculate the Eq. (18), which is given by

$$\begin{aligned} &\min_{g,\mathbf{r},\hat{\mathbf{H}}} g, \quad \text{s.t.} \quad 1^{\mathrm{T}}\mathbf{r} \le g, \quad \hat{h}_n^{\circ} \le r_n \\ &\left\|\mathbf{E}_s\mathbf{W}_{opt} - \hat{\mathbf{A}}\hat{\mathbf{H}}\right\|_{\mathrm{F}} \le \delta, \quad n = 1, 2, \cdots, N, \end{aligned} \tag{19}$$

where vector $\mathbf{r} = [r_1, r_2, \cdots, r_N]$. According to Eq.(15), we can know that the subspace fitting error is $\sqrt{\mathbf{Z}(\boldsymbol{\theta}_0)}$. And the function $(2L/\sigma^2)\mathbf{Z}(\boldsymbol{\theta}_0)$ at the true DOA vector $\boldsymbol{\theta}$ is asymptotically chi-square distributed with $2P'(M-P)$ degrees of freedom, which can be evaluated [22]. Then the regularization parameter δ can be computed by $\sqrt{\mathbf{Z}(\boldsymbol{\theta}_0)} \le \delta$ with a high probability $\rho = 0.99$. In the end, the Eq.(19) can be completed by using the convex optimization toolbox CVX in MATLAB.

IV. Simulation Results

In this part, simulations are used to compare the following different methods with the proposed novel block sparse recovery method by utilizing the WSF framework for DOA estimation. The methods in [12], [16], [17] [18] and [19]

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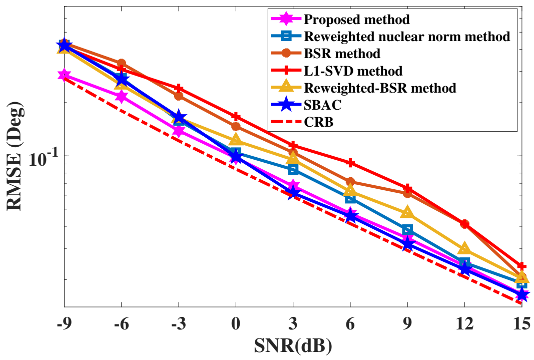

Fig. 3. RMSE versus SNR for coherent signals

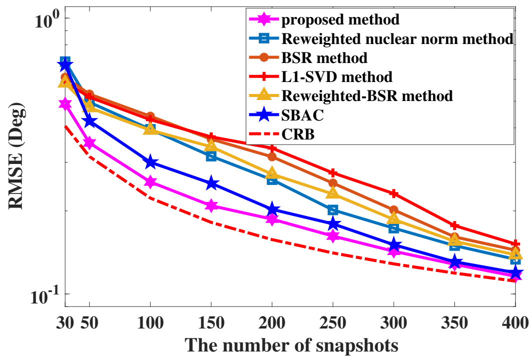

Fig. 5. RMSE versus the number of snapshots for coherent signals

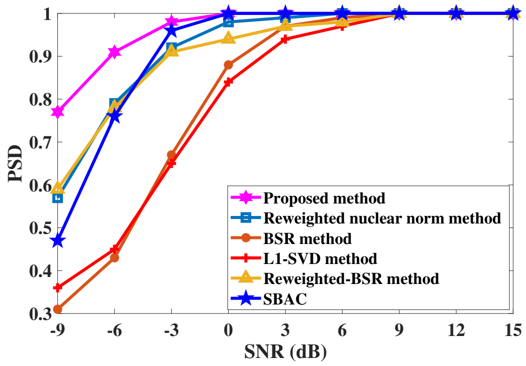

Fig. 4. PSD versus SNR for coherent signals

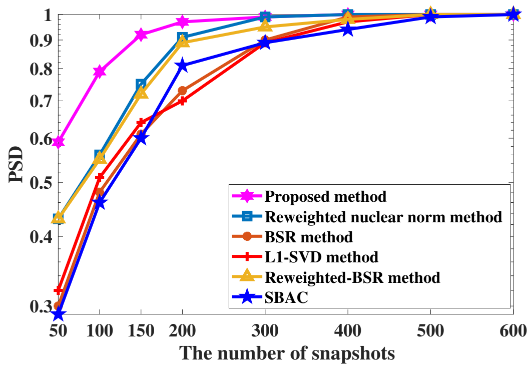

Fig. 6. PSD versus the number of snapshots for coherent signals

are named as reweighted-BSR method, l_1-SVD method, BSR method, the reweighted unclear norm method and SBAC method, respectively. The CRB in [23] is also used to evaluate the performance with above different methods. In all the simulation experiments, the non-zero mutual coupling coefficients are set as $t_1 = 0.1545 - j0.4776$, which indicates that the number of mutual coupling coefficients is $K = 2$. And we suppose that there are a ULA equipped with $M = 10$ antennas, the adjacent sensor is separated by half-wavelength. Then we firstly discrete the whole spatial domain, whose space varies from -90° to 90° with an interval of 0.1°, and then it is efficient to use the adaptive grid method around the estimated peaks. Thus, the estimation precision may be improved while the computation amount is reduced. Meanwhile, as a criterion to measure the DOA estimation performance, the root mean-square error (RMSE) could be written as the follows:

$$\text{RMSE} = \sqrt{\frac{1}{JP}\sum_{i=1}^{J}\sum_{p=1}^{P}(\theta_{p,i} - \theta_p)^2}, \tag{20}$$

where θ_p represents the desired DOA value and the $\theta_{p,i}$ is the estimated DOA value of the pth signal in the ith Monte Carlo experiment. And the number of Monte Carlo experiments is defined as $J = 200$ in all and P represents the number of targets.

Fig. 1 and Fig. 2 display the spatial spectrum with different methods and the RMSE versus SNR for incoherent closely spaced signals, respectively. We assume that the DOAs of the two incoherent closely spaced signal sources are $\theta_1 = -3^\circ$, $\theta_2 = 2^\circ$ and $L = 200$. In Fig. 1, SNR=0dB, and the circle represents the desired DOAs. And as shown in Fig. 1, the proposed method can better distinguish the closely spaced signals than other methods, and with a high degree of accuracy. From Fig. 2, it is obvious to show that the RMSE value of our proposed algorithm is lower than the reweighted nuclear norm method, reweighted BSR method, l_1-SVD method and BSR methods in all SNR ranges and gets much close to CRB. And obviously, the RMSE value of SBAC method is lower than the proposed method in low SNR range(-9dB to 0dB). Thus, the performance of the proposed algorithm is much better than the other algorithms for closely spaced signals, especially in the case of low SNR range.

Fig. 3 and Fig. 4 display the RMSE versus SNR and the PSD versus SNR with different methods, respectively, and the number of snapshots is set as $L = 200$. Assumed that the DOAs of the two coherent signal sources are $\theta_1 = -5^\circ, \theta_2 = 15^\circ$, and the probability of successful detection (PSD) is defined that the gap is less than 0.3° between the true target DOAs and the estimated DOAs in Fig. 4 and Fig. 6. According to the tendency depicted in Fig. 3 and Fig. 4, the proposed method has a higher PSD and lower RMSE value than the reweighted nuclear norm method, reweighted BSR method, l_1-SVD method and BSR methods in all SNR range. And when the SNR is less than 0dB, the estimation performance of the SBAC method is worse than our proposed method. This phenomena is mainly due to the use of the optimal subspace

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fitting framework in our proposed method, which can enhance the sparsity of solution. We can also find out that, as the SNR increases, the PSD of our proposed method reaches 100% first.

Fig. 5 and Fig. 6 show the RMSE versus snapshot number and the PSD versus snapshot number, respectively, and the SNR=-5dB. As the results displayed in Fig. 5, when the number of snapshots increase, the RMSE value of our proposed method is more lower than the rest four methods, which is closer to CRB. And at the same time, the stability of estimation performance and the PSD of all methods are all promoted in Fig. 6. Meanwhile, it is easily to observe that the PSD of our proposed method is all higher than that of the other methods for all snapshot numbers range. Thus, the excellent estimation performance of the method is verified.

V. Conclusion

In this letter, a robust block sparse recovery algorithm by using WSF is presented to estimate DOA with unknown mutual coupling. The proposed method solves the problem of mutual coupling effect and avoids the loss of array aperture, and a novel block sparse recovery framework based on WSF is investigated for DOA estimation. Furthermore, the upper bound of subspace fitting error and an optimal weighted matrix are also given. The experiments have demonstrated that the proposed method can achieve the desired performance with the coherent signals and closely spaced signals.

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